Can be homogenous

From $(a_1 x + b_1 y + c_1)\, dx + (a_2 x + b_2 y + c_2) = 0$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

* نقوم بنقل المحاور إلى النقطة (h, k) نقطة جديدة
فتصبح المعادلة homogenous

$$x = X + h$$
$$y = Y + k$$

$$\therefore \begin{matrix} a_1 h + b_1 k + c_1 = 0 \\ a_2 h + b_2 k + c_2 = 0 \end{matrix} \Rightarrow (a_1 X + b_2 Y)\, dx + (a_2 X + b_2 Y)\, dy = 0$$

ex: 1

$$\frac{dy}{dx} = \frac{4x + 2y - 10}{2x - y - 3}$$

بنقل المحاور إلى (h, k)

$$\therefore 4h + 2k - 10 = 0 \longrightarrow (1)$$

$$2h - k - 3 = 0 \longrightarrow (2)$$

$$\therefore k = 1 \qquad h = 2$$

$$x = X + 2 \qquad y = Y + 1$$

$$\therefore \frac{dy}{dx} = \frac{4X + 2Y}{2X - Y}$$

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$Y = VX \qquad \therefore Y' = V + X\dfrac{dV}{dX}$

$\therefore V + X\dfrac{dV}{dX} = \dfrac{4X + 2VX}{2X - VX}$

$\therefore V + X\dfrac{dV}{dX} = \dfrac{\cancel{X}(4+2V)}{\cancel{X}(2-V)}$

$\therefore X\dfrac{dV}{dX} = \dfrac{4+2V}{2-V} - V$

$\therefore X\dfrac{dV}{dX} = \dfrac{4+\cancel{2V}-\cancel{2V}+V^2}{2-V} = \dfrac{4+V^2}{2-V} =$

$\therefore X\dfrac{dV}{dX} = \dfrac{4+V^2}{2-V} \qquad \therefore \displaystyle\int \frac{1}{X}dX = \int \frac{2-V}{4+V^2}dV$

$\Rightarrow \ln X + C = \displaystyle\int \left(\frac{2}{4-V^2}\right)dV - \int \frac{1}{2}\left(\frac{2V}{4+V^2}\right)dV$

$\ln X + C = 2\tan^{-1}\left(\dfrac{V}{2}\right) - \dfrac{1}{2}\ln(4-V^2)$

$\therefore 2\tan^{-1}\left(\dfrac{Y}{2X}\right) - \dfrac{1}{2}\ln\left(4+\left(\dfrac{Y}{X}\right)^2\right) = \ln X + C$

$X = x-2 \qquad Y = y-1$

$\therefore \Rightarrow 2\tan^{-1}\dfrac{y-1}{2(x-2)} - \dfrac{1}{2}\ln\left(4+\left(\dfrac{y-1}{x-2}\right)^2\right) = \ln(x-2) + C$

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Exact

Form $M(x,y)\,dy + N(x,y)\,dx = 0$

$$\Rightarrow y' = -\frac{N}{M}$$

iff $\Rightarrow$ exact شرط ال

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$$

تكامل الطرفين و حذف المكرر *

ex:- $(y\cos xy + e^{x})\,dy + (x\cos xy - 2y\,e^{y^2})\,dx = 0$

Solution

$M = y\cos xy + e^{x} \quad \therefore \frac{\partial M}{\partial x} = -y^2 \sin xy + e^{x}$

$N = x\cos xy - 2y\,e^{y^2} \quad \therefore \frac{\partial N}{\partial y} = -x^2 \sin xy - 2y \cdot 2y\,e^{y^2} - 2e^{y^2}$

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ex $(y\cos xy + e^{x})\,dx + (x\cos xy - 2y\,e^{y^2})\,dy = 0$

Solution

$M = (x\cos xy - 2y\,e^{y^2})\,dy \quad \therefore \frac{dM}{dx} = -xy\sin xy + \cos xy$

$N = (y\cos xy + e^{x})\,dx \quad \therefore \frac{dN}{dy} = -xy\sin xy + \cos xy$

$\because \frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} \quad \therefore$ exact $\Rightarrow$

Hi . Star

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$\int (y \cos xy + e^{x}) dx + \int (x \cos xy - 2y e^{y^2}) dy = 0$

$\frac{y \sin xy}{y} + e^{x} + \frac{x \sin xy}{x} - 2 \cdot =$

$\sin xy + e^{x} + x \sin xy = 0$

y	e^{y^2}
$\downarrow$	
1	$\frac{e^{y^2}}{2y}$
0	$\frac{e^{y^2}}{4y^2}$

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Exact

Form $M(x,y)\,dx + N(x,y)\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore \text{Exact type}$$

طريقة الحل ⇐ تكامل طرفي المعادلة مع حذف المتكرر

Ex:- $(x^3 + 3xy^2) + (3x^2y + y^3)\frac{dy}{dx} = 0$

$M = x^3 + 3xy^2 \qquad \therefore M_y = 0 + 6xy$

$N = 3x^2y + y^3 \qquad \therefore N_x = 6xy + 0$

$$\therefore M_y = N_x$$

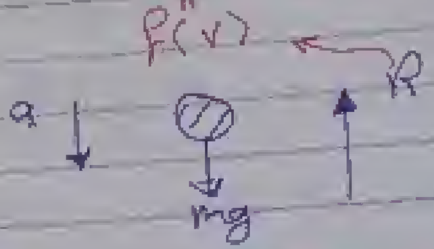
$$\Rightarrow \int (x^3 + 3xy^2)\,dx + (3x^2y + y^3)\,dy = 0$$

$$\frac{x^4}{4} + \frac{3x^2y^2}{2} + 3\,\frac{\cancel{x^3}y^2}{\cancel{2}_3} + \frac{y^4}{4} = C$$

خذ فقط المتكرر

$$\therefore \frac{x^4}{4} + 3\,\frac{x^2y^2}{2} + \frac{y^4}{4} = C$$

① Falling Body



$ma = \Sigma F$

$\therefore \rightarrow ma = mg - R \quad \div m$

مقاومة الجسم اللي ساقط

$\therefore a = g - \frac{R}{m} \qquad \therefore \frac{dv}{dt} = g - \frac{R}{m}$

$\therefore a \nearrow \frac{dv}{dt} \qquad \searrow v\frac{dv}{dy} \rightarrow$ لما يطلب أقصى ارتفاع

$\therefore$ ex:- $R = mkv$ (سرعة v) $\quad (m, k, \text{Constant})$

$\frac{dv}{dt} = g - \frac{mkv}{m} \qquad \therefore \frac{dv}{dt} = g - kv$

$\Rightarrow \frac{dv}{g - kv} = dt \qquad \therefore t = \left(\int \frac{dv}{g - kv}\right)$

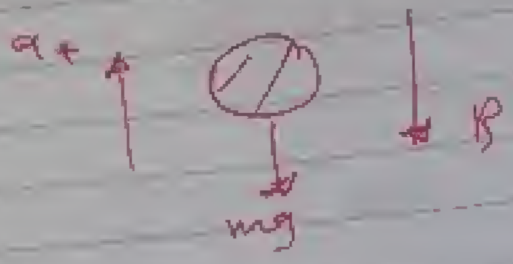
$\therefore t = \frac{1}{-k} \int \frac{-k}{g - kv}\, dv \qquad \therefore \frac{-1}{k} \ln(g - kv) = t + c$

لحظة السقوط

$\Rightarrow$ at $v = 0 \quad t = 0 \qquad \therefore \boxed{\frac{-1}{k} \ln(g) = c}$

$$\therefore \boxed{\frac{-1}{k} \ln(g - kv) + \frac{1}{k} \ln(g) = T}$$

$ma = -(mg + R)$



$a = -\left(g + \frac{R}{m}\right)$

$\rightarrow$ at $T = 0 \quad V = V_0$

[2] Cooling temperature :-

تغير الحرارة بالنسبة للزمن $\frac{dT}{dt} = -K(T - T_m)$ قانون نيوتن للتبريد.

T_m ↓ الوسط المحيط

ex:- a metal bar at temperature of $\boxed{100 F^\circ}$ is placed in a room at constant temp of $\boxed{0^\circ F}$ is after $\boxed{20 \text{ min}}$ $\boxed{T = 50^\circ F}$ find

a- t for $25^\circ F$??

b- T after 10 min ?

Solution

$\frac{dT}{dt} = -K(T - 0) \quad \therefore \frac{dT}{dt} = -KT$

$\therefore \frac{1}{T} dT = -K\,dt$

$\therefore \ln T = -Kt + C_1 = \ln A$

$\therefore T = C e^{-kt}$

$\Rightarrow$ at $t = 0$ $\quad T = 100\,F^{\circ}$ $\quad \therefore 100 = C e^{-0} =$

$\therefore \boxed{C = 100}$

$\therefore T = 100\, e^{-kt}$

$\Rightarrow$ at $t = 20$ mi $\quad T = 50^{\circ} f$

$\therefore 50 = 100\, e^{-k \times 20}$ $\quad \therefore -20k = \ln\left(\frac{1}{2}\right)$

$\therefore k = .0346$

$\therefore T = 100\, e^{-.0346\, t}$

(a) $\rightarrow$ $50 = 100\, e^{-.0346\, t}$ $\quad \therefore t =$

(b) $\rightarrow$ $T = 100\, e^{-.0346 \times 10}$ $\quad \therefore T =$

3 orthogonal Tragectories :-

$\rightarrow$ Given معادلة منحنى

$\rightarrow$ Req معادلة منحنى جديد ⊥ على المنحنى المعطى

$\rightarrow$ Solution ١) إيجاد المعادلة التفاضلية للمنحنى المعطى

$$y'_2 = -\frac{1}{y'_1}$$

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ex:- 1 find the O.T of the family of curves $x^2+y^2=c^2$??

Solution

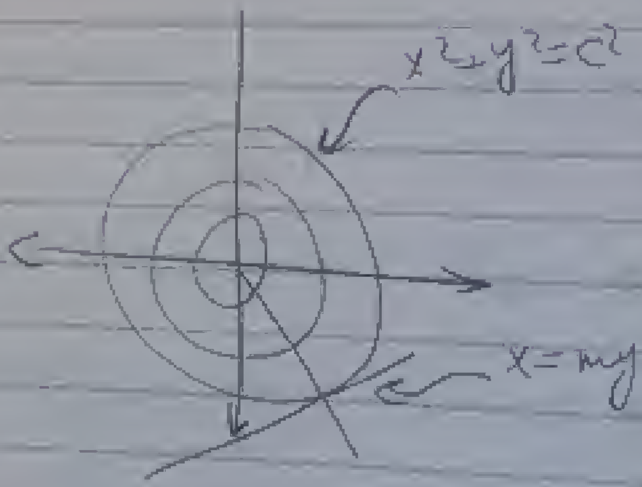
$x^2+y^2=c^2 \quad \therefore 2x+2yy'=0$

$\therefore y'=\frac{-x}{y} \quad \because y_2'=\frac{-1}{y'}$

$\frac{-1}{y'}=\frac{x}{y} \quad \therefore \frac{1}{\frac{dy}{dx}}=\frac{x}{y}$

$\therefore \int \frac{1}{x}dx=\int\frac{1}{y}dy \quad \ln x=\ln y+\ln m$

$\therefore x=my$



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(2) find the o.t of the family of Curves
$x^2 - y^2 = c^2$

Solution

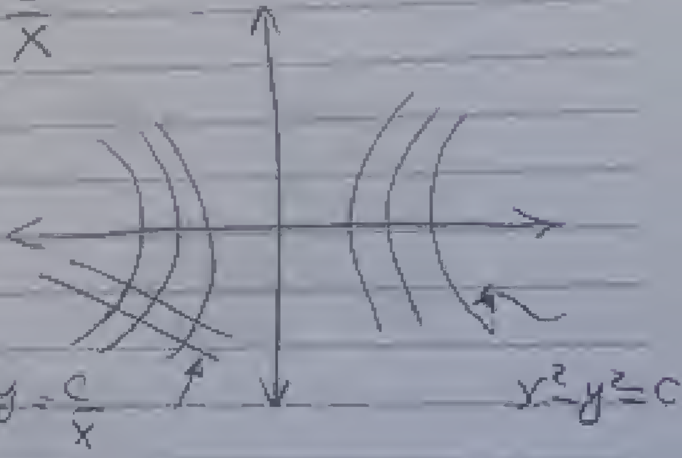
$2x - 2yy' = 0$

$\therefore y' = \frac{2x}{2y} = \frac{x}{y}$ $\therefore \frac{-1}{y'} = \frac{x}{y}$

$\therefore \frac{-1}{\frac{dy}{dx}} = \frac{x}{y}$ $\ln y = -\ln x + \ln c$

$\ln y + \ln x = \ln c$

$\therefore y = \frac{c}{x}$



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(2) find the ot of the family of curves
$x^2+y^2=2by \Rightarrow b=\frac{x^2+y^2}{2y}$

Solution

$2x+2yy'=2by' \rightarrow x+yy'=\left(\frac{x^2+y^2}{2y}\right)y'$

$\therefore x = y'\left(\frac{x^2+y^2}{2y}-y\right)$

Put $y'=\frac{-1}{y'}$

$x=\frac{-1}{y'}\left(\frac{x^2+y^2}{2y}-y\right) = \left(\frac{x^2+y^2-2y^2}{2y}\right)y'$

$\therefore \frac{2xy}{x^2-y^2}=y' \longrightarrow$ homogenous function

$\therefore \frac{-1}{y'}=\frac{2xy}{x^2-y^2} \quad \therefore \frac{y^2-x^2}{2xy}=\frac{dy}{dx}$

$\Rightarrow$ Put $y=vx \qquad y'=\frac{dv}{dx}x+v$

$\frac{v^2x^2-x^2}{2vx^2}=\frac{dv}{dx}x+v$

$\therefore \frac{v^2-1}{2v}-v=x\frac{dv}{dx} \qquad \therefore \frac{v^2-1-2v^2}{2v}=x\frac{dv}{dx}$

$\therefore \frac{-(1-v^2)}{2v}=x\frac{dv}{dx}$

$$\therefore \int -\frac{1}{x}\,dx = \int \frac{2v}{1+v^2}\,dv$$

$$-\ln x + \ln C = \ln(1+v^2)$$

$$\therefore C = x(1+v^2)$$

$$C = x\left(1+\left(\frac{y}{x}\right)^2\right)$$

$$C = \left(\frac{y^2+x^2}{x^2}\right)x \qquad \boxed{Cx = y^2+x^2}$$

[illegible]

$$x^2+y^2=2by \qquad \therefore x^2+y^2-by+b^2-b^2=0$$

$$x^2+(y^2-by+b^2)=b^2$$

$$x^2+(y-b)^2=b^2$$

$$\Rightarrow x^2+y^2=Cx$$

$$x^2+y^2-Cx+\frac{C^2}{4}-\frac{C^2}{4}=0$$

$$\therefore y^2+\left(x-\frac{C}{2}\right)^2=\left(\frac{C}{2}\right)^2$$